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EDUCATIONAL VALUES OF GEOMETRY.

BY F. F. DECKER.

It is doubtless quite unnecessary to point out to an audience of members of the Association of Teachers of Mathematics in the Middle States and Maryland the nature of mathematics or in particular of Euclidean geometry; yet because of its importance as a foundation for an intelligent discussion of methods of teaching geometry we shall refer briefly to it.

Mathematics is, not to attempt too great a refinement, the science of implications; and its work therefore consists in formulating postulates and tracing the inevitable conclusions. Euclidean geometry, the first geometry for the race and for the student, is characterized, in addition to its indefinables and the laws of deduction which it postulates, by its own postulates. The work of the student is to comprehend them and to make various chains of deductions. To perform this work is to study geometry.

Geometry as a science has then no connection with the phenomena about us—is not a natural science. But to assist the student to master the subject the teacher may find it necessary, yes, will many times find it necessary, to refer to the child's sense perception. In forming the concept of Euclidean space one will probably make a comparison with physical space. Thus if free mobility is to be postulated, the difficulty of moving a physical body without changing its size on account of variation of pressure may be illustrated by the case of a balloon sinking in the ocean. Again the difficulty of moving a physical body to any position on account of the impenetrability of physical matter may be pointed out. The former illustration (concerning change in size due to pressure) will suggest to the teacher that a geometry may be constructed containing the postulate that the size of the figure decreases in proportion to the distance it is moved. The successful teacher's mental storehouse will contain many such comparisons to be used when needed, but they will not be considered as geometry.

Again all teachers will have the pupil draw figures, but will consider what seems to happen on the figure as drawn as a matter to be investigated rather than as a deduction. For example in the usual proof of the proposition that every point in a plane equi-distant from two points A and B is on the perpendicular bisector of the line segement joining them by pointing out the inconsistency in assuming that such a point C may not be, one of the lines CA or CB in the figure as drawn seems to cross the perpendicular bisector of AB.

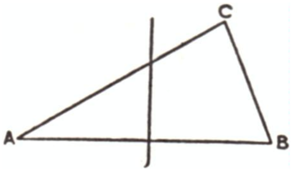

Fig. 1.

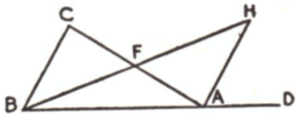

Fig. 2.

As another instance, in the proof of the theorem that an exterior angle of a triangle ABC is greater than either opposite interior angle, says B, the line segment drawn from the vertex A to the end of the line segment BH drawn to the middle of the opposite side and then produced its own length—the line segment AH—seems to be within the exterior angle so as to make the angle HAF which is the equal of ABC a part of DAC.

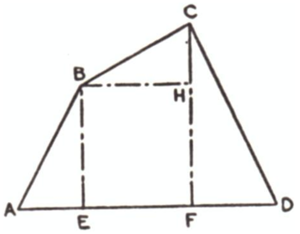

Fig. 3.

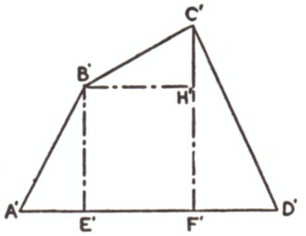

Fig. 4.

We shall give as a third instance what may appear to be a proof of the theorem that if two quadrilaterals have three sides of the one equal to three sides of another, each to each, and the two corresponding angles adjacent to the fourth sides equal, each to each, the quadrilaterals themselves are equal.

Let the given quadrilaterals $ABCD$ and $A'B'C'D'$ have $AB = A'B'$, $BC = B'C'$, $CD = C'D'$, $\angle DAB = \angle D'A'B'$ and $\angle CDA$

$= \angle C'D'A'$. Draw BE and $CF \perp$ to AD, $B'E'$ and $C'F' \perp A'D'$, $BH \perp CF$ and $B'H' \perp C'F'$.

Proof: $\left.\begin{array}{l}\triangle\ BAE = \triangle\ B'A'E', \\ \text{and } \triangle\ CDF = \triangle\ C'D'F'\end{array}\right\}$ Right $\triangle$ with hypotenuse and acute $\angle$ equal.

$\therefore\ BE = B'E'$, $CF = C'F'$, and $CH = C'H'$,

$\therefore\ \triangle\ BHC = \triangle\ B'H'C'$, $\therefore\ BH = B'H'$,

$\therefore\ BHFE = B'H'F'E'$,

$\therefore\ BAE + CDF + BHC + BHFE = B'A'E' + C'D'F' + B'H'C' + B'H'F'E'$, or, as the whole equals the sum of its parts,

$$ABCD = A'B'C'D'.$$

That an assumption has been made in this proof that something that seems to happen in the figure necessarily happens may be seen by a simple method of showing the falsity of the conclusion. The following construction due to Dr. R. L. Moore will show that two quadrilaterals satisfying the hypothesis may be constructed as unequal as we like.

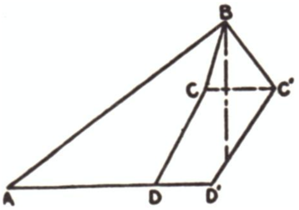

FIG. 5.

Produce a line segment AD making it any greater length, AD'. Draw angles ADC and $AD'C'$ equal. Take DC and $D'C'$ equal and take B on the perpendicular bisector of CC'. The quadrilaterals $ABCD$ and $ABC'D'$ satisfy the hypothesis without being equal.

The last instance will illustrate the fact that geometry knows no necessity except the logical and will commend to the teacher the advisability of putting the other two on logical foundations. But how? Shall we complicate the machinery by deducing the second from a postulate underlying the first or shall we have a postulate for each. Geometry, wincing under a thrust at one of its real charms, cries out for the former; but pedagogy, unlike geometry, has other necessities than the logical, and perhaps classroom experience will dictate a compromise. In this we will agree, that the teacher who substitutes in place of a rational process the observation that certain lines seem to occupy certain positions in a drawn figure is not at that point teaching mathematics. It may be noted in passing that the assumption of Figs. 1 and 2 are neither essentially true nor essentially false, for falsity is a matter of inconsistence and until a fundamental such as Pasch's assumption is introduced, there is nothing with which they can be inconsistent.

True it is that the question is often raised if real geometry can actually be taught to the student in the high school. A society of mathematics teachers like ours, with a high regard for the real beauty of the science, with a keen appreciation of its cultural aspect, will hesitate to deny it. No teacher will answer in the negative until he has familiarized himself with a geometry foundation, consistent but irreducible, such as that of Hilbert or Veblen. Hilbert himself says that geometry made more nearly rational is thereby made more simple. Halsted—and it seems a pity that the name of a man with so ardent a love for pure geometry should not have appeared in the bibliography of science teaching published during the current year for the American Federation of Teachers of the Mathematical and the Natural Sciences, even if only to make intelligible to the reader the answer made to him by Professor Dewey—Halsted seems to believe that rational geometry can be taught to the high school student. We will perhaps hold such a logical system as purely an ideal toward which we will endeavor to make steady advance. Such men cannot be ignored and if we are at any point to teach something else for geometry these men must be answered. The really important consideration seems to be that we make the effort to approach the standard set by the geometer.

And why? Because, for one reason, the opportunity to teach geometry at all is at stake. But why should geometry be taught at all? We may answer: because of the beautiful deductive system it puts before the student; because of the stimulus this system furnishes to the systematizing of bodies of knowledge in other fields. We must be in a position to answer: because of the power the student acquires in framing hypotheses, in sizing up situations, and in deducing conclusions. These things the student must be asked to do throughout the course. These things he must do in life; or rather we would say, the potentiality for training, to a greater or less degree in these directions forms a material part of his birthright.

Says Clifford: "This book (Euclid) has been for nearly twenty-two centuries the encouragement and guide of that scientific thought which is one thing with the progress of man from a worse to a better state. The encouragement; for it contained a body of knowledge that was really known and could be relied on. The guide; for the aim of every student of every subject was to bring his knowledge of that subject into a form as perfect as that which geometry had attained."

A list of concomitant benefits of the study of geometry of considerable length might be given, including such as skill in mechanical drawing, development of the space intuition, of accuracy in computation, and the application to physical space; but we are considering those benefits that have their source in the nature of the subject itself and we mention only these: the appreciation of a type of the beautiful and the useful, the stimulus to follow the model and the ability so to do.

Geometry teaching suffers from the fact that the subject seems to have a practical application. When a teacher becomes weary of the great task of assisting pupils through this system of deductive reasoning he is exposed to the danger of forsaking his real work for that of some practical application. He will say, perhaps, that his pupil is going to enter some trade and is in most need of specialized manual skill. What a pity! For if one student more than another needs to be able to think it is the one whose environment is in the greatest need of improvement.

Not long since the writer was privileged to hear a discussion of the advisability of establishing a vocational high school, led by a manufacturer and a journeyman tradesman. The manufacturer dwelt on the necessity of every man earning a living, of the necessity of getting recruits for the trades, and the advantage resulting from the co-operation between the factory

(*To be continued.*)